EXERCISE – II HINTS & SOLUTIONS

Sol.1 C,D

$$\int e^{3x} \cos 4x \, dx = e^{3x} (A \sin 4x + B \cos 4x) + c$$

Diff. both the side w.r.t. x

$e^{3x} \cos 4x = 3e^{3x} (A \sin 4x + B \cos 4x)$
$+ e^{3x} (4A \cos 4x - 4B \sin 4x)$

$\cos 4x = (3A \sin 4x + 3B \cos 4x) + 4A\cos 4x - 4B\sin 4x$

$1 = 3B + 4A$

$3A - 4B = 0$

$3A = 4B$

Sol.2 A,B

$$\int \frac{dx}{5 + 4\cos x} dx$$

$$= \int \frac{\sec^2 \frac{x}{2} dx}{5\left(1 + \tan^2 \frac{x}{2}\right) + 4\left(1 - \tan^2 \frac{x}{2}\right)}$$

$\tan \frac{x}{2} = t \Rightarrow \sec^2 \frac{x}{2} dx = 2dt$

$$= 2\int \frac{dt}{9 + t^2} \qquad = \frac{2}{3} \tan^{-1}\left(\frac{1}{3} \tan \frac{x}{2}\right) + c$$

$$I = \frac{2}{3}, \; m = \frac{1}{3}$$

Sol.3 B,C,D

$$\int \frac{x^2 + \cos^2 x}{1 + x^2} \operatorname{cosec}^2 x \, dx$$

$$= \int \frac{x^2 + 1 - 1 + \cos^2 x}{1 + x^2} \operatorname{cosec}^2 x \, dx$$

$$= \int \operatorname{cosec}^2 x \, dx - \int \frac{dx}{1 + x^2}$$

$= -\cot x - \tan^{-1} x + c$

$$= -\cot x - \left(\frac{\pi}{2} - \cot^{-1} x\right) + c$$

$= -\cot x + \cot^{-1} x + C$

$= -\cot x - e^{\ell n \tan^{-1} x} + C$

Sol.4 A,B,C,D

$$\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$$

$$= \int \frac{2 \sin x \cos x}{\sin^4 + \cos^4 x} dx$$

$$= \int \frac{2 \tan x \sec^2 x}{1 + \tan^4 x} dx$$

put $\tan^2 x = t \Rightarrow 2 \tan x \sec^2 x \, dx = dt$

$$= \int \frac{dt}{1 + t^2} = \tan^{-1} (\tan^2 x) + c$$

$= -\cot^{-1} (\tan^2 x) + c'$

$= \cot^{-1} (\cot^2 x) + c''$

$= -\tan^{-1} \cos 2x + c$

$$= -\tan^{-1}\left(\frac{1 - \tan^2 x}{1 + \tan^2 x}\right) + c$$

$= -(\tan^{-1} 1 - \tan^{-1} (\tan^2 x) + c$

$= \tan^{-1} (\tan^2 x)) + C$

Sol.5 A,B

$$\int \frac{dx}{\sqrt{x - x^2}}$$

put $\quad x = \sin^2 \theta \Rightarrow dx = \sin 2\theta \, d\theta$

$$= \int \frac{\sin 2\theta \, d\theta}{\sqrt{\sin^2 \theta - \sin^4 \theta}}$$

$$= \int \frac{2 \sin \theta \cos \theta \, d\theta}{\sin \theta \cos \theta}$$

$= 2\theta + c = 2 \sin^{-1} \sqrt{x} + c$

$$= 2\left(\frac{\pi}{2} - \cos^{-1}\sqrt{x}\right) + c$$

$= c' - 2 \cos^{-1} \sqrt{x}$

$= c' - \cos^{-1} (2x - 1)$

$= \sin^{-1} (2x - 1) + C$

Sol.6 B,D

$$\int \frac{\ell n\left(\frac{x - 1}{x + 1}\right)}{(x^2 - 1)} dx$$

Let $\left(\frac{x-1}{x+1}\right) = t \Rightarrow \frac{dx}{x^2-1} = \frac{dt}{2}$

$= \frac{1}{2}\int t\,dt = \frac{t^2}{4} + c$

$= \frac{1}{4}\,\ell n^2\left(\frac{x-1}{x+1}\right) + c$

$= \frac{1}{4}\,\ell n^2\left(\frac{x+1}{x-1}\right) + c$

Sol.7 A,C,D

$\int \frac{\ell n\,(\tan x)}{\sin x.\cos x}\,dx$

put ℓn (tan x) = t $\Rightarrow \frac{1}{\tan x}$. $\sec^2 x\,dx = dt$

$\frac{dx}{\sin x\cos x} = dt$

$= \int t\cdot dt = \frac{t^2}{2} + c$

$= \frac{\ell n^2\,(\tan x)}{2} + c = \frac{\ell n^2\,(\cot x)}{2} + c$

$= \frac{1}{2}\,\ell n^2\,(\sin x\sec x) + c$

Sol.8 A,C

$\int \frac{x-1}{x^2\sqrt{2x^2-2x+1}}\,dx = \frac{\sqrt{f(x)}}{g(x)} + c$

put x = 1/t $\Rightarrow dx = -\frac{dt}{t^2}$

$= \int \frac{\frac{1}{t}-1}{\frac{1}{t^2}\sqrt{\frac{2}{t^2}-\frac{2}{t}+1}}\left(-\frac{dt}{t^2}\right)$

$= \int \frac{(t-1)\,dt}{\sqrt{t^2-2t+2}} = \frac{1}{2}\int \frac{(2t-2)\,dt}{\sqrt{t^2-2t+2}}$

Let $t^2 - 2t + 2 = z^2 \Rightarrow (2t-2)\,dt = 2z\,dz$

$= \frac{1}{2}\int \frac{2z\,dz}{z}$

$= z + c$

$= \sqrt{t^2-2t+2} + c$

$= \sqrt{\frac{1}{x^2}-\frac{2}{x}+2} + c$

$= \frac{\sqrt{2x^2-2x+1}}{x} + c$

g(x) = x

$f(x) = 2x^2 - 2x + 1$

Sol.9 A,C

$\int \frac{\ell n\,(x+1)-\ell n\,x}{x(x+1)}\,dx$

$\int \frac{\ell n\left(\frac{x+1}{x}\right)}{x(x+1)}\,dx$

Let $\ell n\left(\frac{x+1}{x}\right) = t \quad \frac{1}{\left(\frac{x+1}{x}\right)}\,\frac{x-(x+1)}{x^2}\,dx = dt$

$\frac{dx}{x(x+1)} = -\,dt$

$= -\int t\,dt = -\frac{t^2}{2} + c$

$= -\frac{1}{2}\left[\ell n^2\left(\frac{x+1}{x}\right)\right] + c$

$= -\frac{1}{2}\,[\ell n\,(x+1) - \ell n\,x]^2 + c$

$= -\frac{1}{2}[\ell n^2\,(x+1) + \ell n^2\,(x) - 2\,\ell n\,x\,.\,\ell n\,(x+1)] + c$

$= -\frac{1}{2}\,\ell n^2\,(x+1) - \frac{1}{2}\,\ell n^2\,(x) + \ell n\,x\,\ell n\,(x+1) + c$

$= -\frac{1}{2}\,\ell n^2\left(1+\frac{1}{x}\right) + c$